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Reserve Requirements and the Inflation Tax

ALTHOUGH MOST MODELS OF INFLATIONARY FINANCE consider only the inflation tax on currency (fiat money), governments typically levy the inflation tax on non-interest-bearing required reserves of the banking system as well as on currency held by the public. Two recent papers by Walsh (1984) and Romer (1985) have provided general equilibrium frameworks that are potentially capable of characterizing inflationary finance on currency and bank deposits. Since the reserve requirement acts as a tax on the intermediation services performed by a bank, any analysis of the reserve requirement must specify the special characteristics of banks' liabilities or assets that allow a positive reserve ratio to coexist with a nonzero level of banking activity. Walsh places bank deposits and currency into a cash-in-advance constraint, while Romer places currency and deposits directly into a representative agent's utility function. Both formulations therefore motivate the existence of the banking sector by stressing the characteristics of bank deposits that differ from the characteristics of liabilities of other nonbank financial intermediaries.[1]

This paper incorporates reserve requirements into a general equilibrium setting along the lines of Walsh and Romer. However, rather than specify a cash-in-advance constraint or place money and deposits into a representative agent's utility

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[1] Williamson (1986) provides an alternative framework in which the general equilibrium impact of reserve requirements can be assessed by stressing the role of the banking system in monitoring debt contracts when the investment process is characterized by asymmetric information.

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function, this paper introduces banks into the model by specifying a transactions technology in which currency and bank deposits permit agents to economize on the amount of time spent on transacting in the goods market. The use of a "shopping-time" technology to motivate the demand for currency has been used recently by Drazen (1979), Fischer (1983), McCallum (1983), and Kimbrough (1986). This paper generalizes the currency-based shopping technology by specifying a transactions technology that is convex in currency and demand deposits so that currency and demand deposits are substitutes in allowing the representative agent to economize on time spent transacting. Since many standard issues of inflationary finance are precluded by the unitary velocity of money implied by standard cash-in-advance constraints, the use of a technology that conserves on transacting time is a useful way to motivate the demand for money without imposing a unitary velocity restriction. The transactions technology of the paper is also consistent with Romer's money-and-deposits-in-the-utility-function model, provided that all transactions time comes out of leisure. In the more general case considered in this paper the use of the inflation tax can also have output effects since an increase in time spent on transacting will generally cause the agent to reduce time spent producing.

As shown in Section 1, the transactions technology specified in this paper has the attractive property that utility-maximizing behavior generates standard asset demand functions in which the demands for money and deposits depend on the opportunity cost of holding each asset and on the level of consumption. Section 2 uses the asset demand functions to develop the characteristics of inflationary finance in the model. The analysis shows that the traditional use of the semielasticity of demand for real money balances with respect to the nominal interest rate to calculate the revenue-maximizing nominal interest rate may seriously underestimate the true revenue-maximizing rate. Section 3 develops the welfare consequences of the use of the reserve ratio and nominal interest rate. The section demonstrates that a monetary authority that minimizes the welfare cost of inflationary finance will generally alter both the reserve ratio and the nominal interest rate in response to changes in revenue requirements from the inflation tax. Section 4 concludes.

1. THE MODEL

The economy is inhabited by a combined representative agent and bank whose preferences over consumption (c) and leisure (ℓ) are given by the concave utility function $U(c, \ell)$. Consumption and leisure are both assumed to be normal goods. The agent produces a consumption good using a production technology that is linear in labor inputs (n). In order to purchase the consumption good, the agent must engage in shopping activity (s). The time spent shopping is reduced by the use of a twice-differentiable transactions technology that is convex in real balances of currency (m) and demand deposits (d): $s = \phi(m, d)c$, where ϕ_m, $\phi_d < 0$; ϕ_{mm}, ϕ_{dd}, $\phi_{md} > 0$.[2] Currency and demand deposits are assumed to be normal goods,

[2]This form of the transactions technology implies that currency and demand deposits are substitutes in conducting transactions and is based on the cash-and-deposits-in-advance constraint used by Walsh.

implying that $\phi_d\phi_{mm} - \phi_m\phi_{md} < 0$, $\phi_m\phi_{dd} - \phi_d\phi_{md} < 0$. The total time spent producing, shopping, and enjoying leisure sums to one. Currency pays a zero nominal interest rate while demand deposits pay the nominal interest rate i^d.

The government finances lump-sum transfers (z) to the agent by levying the inflation tax on the real monetary base, $h \equiv m + \tau d$, where τ is the legal reserve requirement on demand deposits. Legal reserves, like currency, pay a zero nominal interest rate. The government will be assumed to follow a time-consistent monetary policy that prevents price level jumps. As Auernheimer (1974) first showed, such a policy implies that steady-state revenue from the inflation tax will equal $i(m + \tau d)$, where i is the nominal interest rate.

Because there are no inherent dynamic issues connected with the agent's maximization problem, the paper will consider only a steady-state characterization of the agent's problem. In the steady state the economy's real interest rate is determined by the agent's rate of time preference so that the monetary authority determines the nominal interest rate. Bank loans pay the nominal rate of interest. By assuming competitive and costless banking, the nominal rate on deposits will equal one minus the reserve ratio times the nominal interest rate so that the difference between the nominal interest rate and the demand deposit rate is the tax wedge $i\tau$ on intermediation services of the bank.

Equation (1) gives the agent's maximization problem, where the agent's budget constraint incorporates inflation tax payments and lump-sum transfers from the government:

$$\mathcal{L} = \underset{\substack{c,\, \ell \\ m,\, d}}{\text{Max}}\{U(c, \ell) + \lambda[1 - \ell - \phi(m, d)c - c - i(m + \tau d) + z]\}\,. \quad (1)$$

The agent's maximization problem given in (1) produces the first-order conditions that are given in equations (2) through (4):

$$U_c(c, \ell) = [1 + \phi(m, d)]U_\ell(c, \ell) \quad (2)$$

$$-\phi_m(m, d)c = i \quad (3)$$

$$-\phi_d(m, d)c = i\tau = i - i^d\,. \quad (4)$$

Total differentiation of equations (3) and (4), while holding consumption constant, generates the following asset demand equations for currency and demand deposits:

$$\begin{aligned} m &= m(\underset{-}{i},\ \underset{+}{i - i^d},\ \underset{+}{c}) \\ d &= d(\underset{+}{i},\ \underset{-}{i - i^d},\ \underset{+}{c})\,, \end{aligned} \quad (5)$$

where $m_1 = -\phi_{dd}/D$, $m_2 = \phi_{md}/D$, $m_3 = (-\phi_m\phi_{dd} + \phi_d\phi_{md})/D$; $d_1 = \phi_{md}/D$, $d_2 = -\phi_{mm}/D$, $d_3 = (-\phi_d\phi_{mm} + \phi_m\phi_{md})/D$, and $D = c(\phi_{mm}\phi_{dd} - \phi_{md}^2)$. The asset demand functions show that the demands for currency and demand deposits depend negatively on the opportunity cost of holding each asset and positively on the opportunity cost of holding the other asset. Asset demands are also a positive function of the level of consumption.[3]

2. REVENUE FROM THE INFLATION TAX

Assume that the government wishes to calculate the inflation rate and the reserve ratio that will maximize the government's inflation tax revenue:

$$R(i, \tau) = ih(i, \tau) = i(m + \tau d) . \qquad (6)$$

The government has two tax instruments, the nominal interest rate and the reserve ratio, to determine the opportunity cost of holding currency (i) and the opportunity cost of holding demand deposits ($i - i^d$). Revenue maximization with respect to the nominal interest rate, while holding the reserve ratio constant, produces the following first-order condition:

$$i = -\frac{m + \tau d}{m_1 + m_2\tau + m_3\left(\frac{\partial c}{\partial i}\right) + \tau\left[d_1 + d_2\tau + d_3\left(\frac{\partial c}{\partial i}\right)\right]} = -\frac{h}{\frac{\partial h}{\partial i}} . \qquad (7)$$

Equation (7) indicates that a revenue-maximizing government will set the elasticity of demand for the monetary base $[-(i/h)(\partial h/\partial i)]$ equal to one.[4]

Revenue maximization with respect to the reserve ratio is equivalent to maximizing the size of the monetary base for any given interest rate, the condition for which is shown in (8):

$$\frac{\partial h}{\partial \tau} = m_2 i + m_3\left(\frac{\partial c}{\partial \tau}\right) + \tau d_2 i + \tau d_3\left(\frac{\partial c}{\partial \tau}\right) + d = 0 . \qquad (8)$$

Equations (7) and (8) together determine the revenue-maximizing combination of the reserve ratio and nominal interest rate. Using these two conditions, the revenue-maximizing elasticity conditions for the demand for currency (m) and currency plus demand deposits ($m1$) with respect to the nominal interest rate can be expressed as in (9):[5]

[3] Although consumption is taken as exogenous in the asset demand functions of (5), by making use of all of the first-order conditions for the agent's maximization problem, it can be shown that an increase in the nominal interest rate or the reserve ratio will lower consumption, given that consumption and leisure are normal goods and provided that shopping time is nondecreasing in the level of consumption; i.e., that $\partial s/\partial c = \partial \phi(m, d)c/\partial c \geq 0$.

[4] Friedman (1971) mentions but does not analyze the revenue-maximizing condition of equation (7), while Calvo and Fernandez (1983) analyze the special case of (7) with currency set equal to zero.

[5] The elasticity expressions in (9) are based on the derivatives of the first two arguments of the asset

$$\eta_{mi} = 1 + \frac{h}{m}\left[\eta_{h\tau} - \eta_{hc}(\eta_{ci} - \eta_{c\tau})\right]$$

$$\eta_{m1,i} = 1 - \frac{h}{m1}\left[\frac{1-\tau}{\tau}\,\eta_{h\tau} + \eta_{hc}\left(\eta_{ci} + \frac{1-\tau}{\tau}\,\eta_{c\tau}\right)\right], \tag{9}$$

where $\eta_{mi} = -\frac{i}{m}(m_1 + m_2\tau)$, $\eta_{m1,i} = -\frac{i}{m1}(m_1 + m_2\tau + d_1 + d_2\tau)$, $\eta_{h\tau} = \frac{\tau}{h}\frac{\partial h}{\partial \tau}$, $\eta_{hc} = \frac{c}{h}\frac{\partial h}{\partial c}$, $\eta_{ci} = -\frac{i}{c}\frac{\partial c}{\partial i}$, $\eta_{c\tau} = -\frac{\tau}{c}\frac{\partial c}{\partial \tau}$. The elasticity expressions in (9) are gross elasticities that take into account the total effect of a change in the nominal interest rate on the demand for money, including the cost of financial intermediation. The two expressions in (9) indicate that when the revenue-maximizing reserve ratio is chosen, the elasticity of demand for the monetary base with respect to the reserve ratio will equal zero [from equation (8)] so that the government will set the elasticities of demand for currency and currency plus demand deposits ($m1$) greater than or less than one depending on the relative magnitudes of the elasticities of consumption with respect to the nominal interest rate and the reserve ratio.

If there are no real output effects of the inflation tax—that is, if all shopping time comes out of leisure—then the usual result holds at the point of revenue maximization: the monetary authority will set the elasticity of demand for currency or for currency plus demand deposits with respect to the nominal interest rate equal to one.[6] If, on the other hand, the monetary authority is constrained to require a reserve ratio less than the revenue-maximizing ratio, the elasticity of demand for the monetary base with respect to the reserve ratio will be positive, so that a constrained revenue-maximizing monetary authority will set the elasticity of demand for currency with respect to the nominal interest rate greater than one at the same time that the elasticity of demand for currency plus demand deposits with respect to the nominal interest rate is set less than one.

The intuition behind the constrained use of the nominal interest rate to generate inflation tax revenue is straightforward. With a reserve-ratio constraint, the monetary authority attempts to set two tax rates (on currency and demand deposits) with one tax instrument. Without the use of the reserve ratio, the monetary authority may wish to raise the inflation rate to a level greater than the revenue-maximizing rate for currency if the loss of revenue is less than the incremental revenue generated from the inflation tax on required reserves. Full control over both the reserve ratio

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demand functions with respect to the nominal interest rate. This convention was adopted because empirical estimates of the revenue-maximizing inflation rate always express the demand for money as a function of the nominal interest rate (or expected inflation rate) and consumption (or output), without taking into account the additional effect of the nominal interest rate on consumption. To determine the total revenue-maximizing elasticities in (9), the expressions $\eta_{mc}\eta_{ci}$ and $\eta_{m1,c}\eta_{ci}$ should be added to the right-hand sides of the elasticity conditions.

[6] Siegel (1981) arrived at the same condition in a partial equilibrium setting. Siegel's results correspond (in this paper's model) to a specification of a separable transactions technology (so that $\phi_{md} = 0$) with transactions time coming entirely out of leisure.

and nominal interest rate eliminates the need to tax currency at a rate greater than the revenue-maximizing rate, since the asset substitution away from currency and toward demand deposits that accompanies a higher nominal interest rate can be offset by the use of a higher reserve ratio.[7]

Many empirical studies on hyperinflations, beginning with Cagan (1956), have relied on estimates of the demand for currency or currency plus demand deposits to calculate revenue-maximizing inflation rates. The estimated revenue-maximizing inflation rates are uniformly lower than observed inflation rates. The elasticity expressions in (9) of the total effect of an increase in the nominal interest rate on the demand for money suggest that even accurate empirical estimates of the demand for money will not generally produce accurate estimates of the revenue-maximizing inflation rate. For example, let the transactions cost function take the following form:

$$\phi(m, d) = \frac{m}{\alpha}(\ln m - \alpha_0 - 1) + \frac{d}{\beta}(\ln d - \beta_0 - 1) + K, \tag{10}$$

where $0 < m, d < 1$, and K is a positive constant that is large enough to make $\phi(m, d)$ positive. By making use of the asset demand functions given in equation (5) it is easy to show that (10) implies that asset demands can be written as $\ln m = \alpha_0 - \alpha i$ and $\ln d = \beta_0 - \beta(i - i_d)$. Hence, the semielasticity of the demand for currency with respect to the nominal interest rate is equal to α and the semielasticity of the demand for deposits with respect to the opportunity cost of holding deposits $(i - i_d)$ is equal to β.

With the transactions technology given by (10), the set of points defined by the first-order revenue-maximizing condition with respect to the reserve ratio given in (8) becomes the condition $i\tau = 1/\beta$. The set of points defined by the first-order revenue-maximizing condition with respect to the nominal interest rate given in (9) becomes the condition

[7]The observation that constrained revenue maximization results in different elasticity conditions than unconstrained revenue maximization can be extended to regimes in which a reserve requirement is also placed on time deposits, perhaps because the liquidity of time deposits allows them to enter into the transactions technology via arrangements such as the automatic transfer of savings into checking accounts. Setting aside output effects of the inflation tax, the relevant set of elasticity conditions for revenue maximization with two reserve requirements is the following, where $m1$ is currency plus demand deposits, $m2$ is $m1$ plus time deposits, θ is the reserve ratio on time deposits, and h is the monetary base that is composed of currency plus required reserves on demand deposits and time deposits:

$$\eta_{mi} = 1 + \frac{h}{m}\left[\eta_{h\tau} + \eta_{h\theta}\right]$$

$$\eta_{m1,i} = 1 + \frac{h}{m1}\left[-\frac{1-\tau}{\tau}\eta_{h\tau} + \eta_{h\theta}\right]$$

$$\eta_{m2,i} = 1 - \frac{h}{m2}\left[\frac{1-\tau}{\tau}\eta_{h\tau} + \frac{1-\theta}{\theta}\eta_{h\theta}\right]$$

These elasticity expressions suggest that the calculation or estimation of the revenue-maximizing nominal interest rate is very sensitive to the assumption that the government is also jointly using required reserve ratios on different classes of bank deposits to maximize revenue.

$$i = \frac{1}{\alpha} + \frac{\beta \tau d}{\alpha m} \left[\frac{1}{\beta} - i\tau \right] . \tag{11}$$

When the reserve ratio is zero, the revenue-maximizing nominal interest rate will equal $1/\alpha$. At the point of unconstrained revenue maximization (where $i\tau = 1/\beta$) the revenue-maximizing nominal interest rate will also equal $1/\alpha$ and the revenue-maximizing reserve ratio will equal α/β. If the monetary authority is constrained to use the reserve ratio at a level less than α/β, constrained revenue maximization will cause the monetary authority to set the nominal interest rate greater than $1/\alpha$.

The revenue-maximization problem can be portrayed graphically by drawing the two first-order revenue-maximizing conditions as functions of the two revenue-generating policy instruments. The transactions technology specified in (10) implies that the set of points formed by the first-order revenue-maximizing condition with respect to the reserve ratio is a hyperbola and that the set of points formed by the first-order revenue-maximizing condition with respect to the nominal interest rate is concave with respect to the reserve ratio.

The Appendix shows that this characterization of the first-order conditions for revenue maximization also holds whenever the transactions cost function is quadratic and transactions time comes out of leisure.[8] In the more general case derived in the Appendix, the vertical asymptote for the first-order revenue-maximizing condition with respect to the reserve ratio is the line defined by $\tau = \phi_{md}/\phi_{mm}$. Figure 1 presents the graphical solution to the revenue maximization problem as the point of intersection R^* of the curves defined by the two first-order conditions $\partial R/\partial i = 0$ and $\partial R/\partial \tau = 0$.[9]

Bomberger and Makinen (1983) have already verbally made much the same point as is made by Figure 1 in their analysis of the Hungarian hyperinflation of 1945–46. They provide evidence that it was the existence of partially indexed bank deposits (tax pengö accounts) that produced the extreme severity of the hyperinflation. Since indexing of tax pengö accounts was lagged from one to two days, the inflation rate was raised to a level much greater than the revenue-maximizing rate for currency alone in order to tax the partially indexed bank accounts. Revenue maximization that is constrained by partial indexation of bank accounts is analytically equivalent to revenue maximization with a less-than-revenue-maximizing reserve requirement. In both cases the constrained maximization of revenue will result in the elasticity of the demand for currency set at a value that is greater than the unconstrained revenue-maximizing elasticity.

Bomberger and Makinen (1983) indicated that the constraint imposed on revenue

[8]The assumption that all transactions time comes of leisure simplifies the analytical derivation of Figure 1. Including the negative output effect of the inflation tax serves primarily to shift the $\partial R/\partial i = 0$ schedule downward and the $\partial R/\partial \tau = 0$ curve inward.

[9]Although the transactions technology given by (10) implies that the revenue-maximizing nominal interest rate is equal to $1/\alpha$ both at a zero reserve ratio and at the reserve ratio τ^*, Figure 1 shows the more general case in which the semielasticity of the demand for currency with respect to the nominal interest rate differs at the two reserve ratios. In Figure 1 the $\partial R/\partial i = 0$ schedule reaches a peak between a zero reserve ratio and the revenue-maximizing reserve ratio (τ^*). This result holds for both the semilog asset demand example and for the general quadratic case discussed in the Appendix but need not hold for all transactions cost functions.

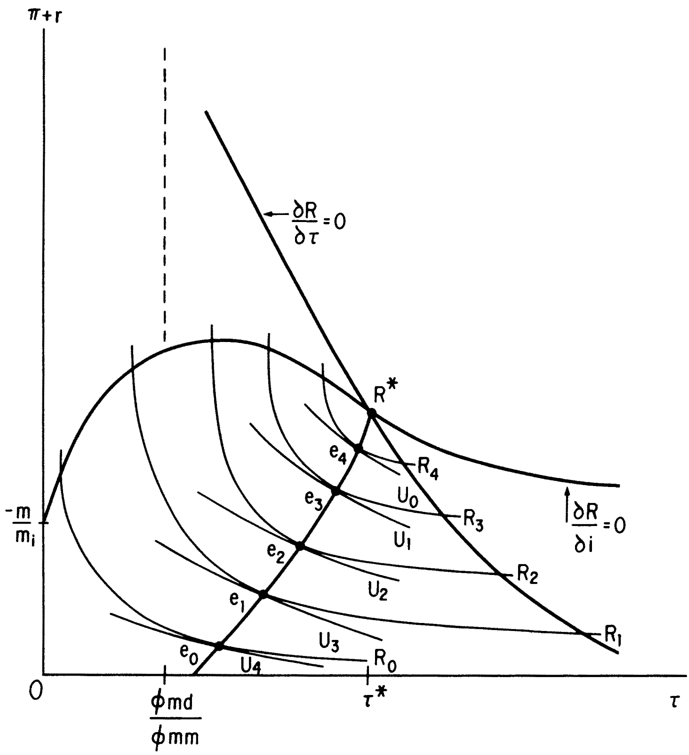

FIG. 1. Revenue from the Inflation Tax as a Function of the Reserve Ratio and Nominal Interest Rate

maximization from the inflation tax in Hungary was related to government resistance to making large reparations payments imposed by the Soviet Union after World War II. In other countries it may be the case that monetary authorities face more binding legal constraints in the discretionary use of reserve requirements than in varying the rate of growth of the monetary base. Such legal constraints in the United States currently limit the Fed to an 18 percent ceiling on reserve requirements on checkable deposits and to a 9 percent ceiling on time and savings deposits. Tamagna (1965) has also documented the widespread incorporation of upper legal limits on reserve requirements in a number of Latin-American countries. In some cases, the constraint that the reserve ratio cannot exceed one may also be binding, as will be the case if α exceeds β for the transactions technology given in (10). If an administration *is* constrained in its use of legal reserve requirements on bank deposits, this section's results emphasize that calculated revenue-maximizing inflation rates that rely on estimates of the demand for currency or currency plus demand deposits as a function of expected inflation or the nominal interest rate may poorly approximate the constrained revenue-maximizing inflation rates.

3. CONSTRAINED WELFARE MAXIMIZATION

In general, governments that rely on the inflation tax do not attempt to maximize revenue. Given that a government must raise a certain amount of revenue with the inflation tax, this section derives the optimal use of the reserve requirement and the inflation rate. If the level of lump-sum transfers is less than the maximum revenue that can be generated from the inflation tax, the government will be able to generate the required revenue with a wide range of combinations of inflation rates and reserve ratios. For any revenue requirement that is less than the maximum revenue, the locus of points found by setting $dR = (\partial R/\partial i)di + (\partial R/\partial \tau)d\tau = 0$ will define an iso-revenue curve. In addition, for any given level of utility of the representative agent, the locus of points found by setting $dU = (\partial U/\partial i)di + (\partial U/\partial \tau)d\tau = 0$ will define an iso-welfare curve facing the government.

Optimal combinations of the reserve ratio and the inflation rate will occur at points of tangency between the iso-revenue and the iso-welfare curves. At these points, the Appendix shows that the elasticity of the demand for currency with respect to the nominal interest rate is the following:

$$\eta_{mi} = 1 + \frac{h}{m}\left[\left(1 - \frac{\eta_{Ui}}{\eta_{U\tau}}\right)\eta_{h\tau} - \eta_{hc}\left(\eta_{ci} - \eta_{c\tau}\right)\right] . \tag{12}$$

where $\eta_{Ui} = -\frac{i}{U}\frac{\partial U}{\partial i}$, $\eta_{U\tau} = -\frac{\tau}{U}\frac{\partial U'}{\partial \tau}$, and $\frac{\eta_{Ui}}{\eta_{U\tau}} = 1 + \frac{\phi_m\phi_{dd} - \phi_d\phi_{md}}{\tau(\phi_d\phi_{mm} - \phi_m\phi_{md})}$ > 1 by the assumption that money and demand deposits are normal goods. A comparison of (12) and (9) shows that a welfare-maximizing government that is unconstrained in its use of the reserve ratio and nominal interest rate will *always* set the gross elasticity of demand for currency with respect to the interest rate to a value that is less than one, even though the revenue-maximizing elasticity may exceed one at the chosen reserve ratio. Equations (9) and (12) together also indicate that the only condition under which a welfare-maximizing government will set the elasticity of demand for currency greater than one is if the government is constrained in its use of the reserve requirement.

The constant-revenue trade-off between the reserve ratio and the nominal interest rate is shown in Figure 1 by the five downward-sloping iso-revenue curves labeled R_0 through R_4. The curves are vertical where they cross the $\partial R/\partial i = 0$ schedule and horizontal where they cross the $\partial R/\partial \tau = 0$ schedule. The government's trade-off between the nominal interest rate and reserve ratio combinations that maintain the agent at a constant level of welfare is represented by the downward-sloping curves labeled U_0 through U_4. As the Appendix shows, these curves have the slope $-i(\phi_d\phi_{mm} - \phi_m\phi_{md})/[\tau(\phi_d\phi_{mm} - \phi_m\phi_{md}) + (\phi_m\phi_{dd} - \phi_d\phi_{md})] < 0$, by the normality assumption on currency and demand deposits.

The set of welfare-maximizing combinations of the reserve ratio and the nominal interest rate, given the government's revenue constant, connect the horizontal axis and the revenue-maximizing point R^* via the tangency points e_0 through e_4. The tangency points are the graphical representation of equation (12). The slope of the locus of tangency points will depend on the functional form of the transactions technology. With the transactions technology given by equation (10), it is straightforward to show that the locus of tangency points is vertical. In an economy characterized by such a transactions technology, a monetary authority that attempts to minimize the welfare cost of inflationary finance will choose the unique reserve ratio $\tau = \alpha / \beta$ and vary the inflation rate as revenue requirements change. An alternative transactions technology is the following: $\phi(m, d) = m^{-\mu} + d^{-\nu}$. With such a technology the slope of the set of tangency points between the iso-revenue and the iso-welfare curves will equal $i\nu / (\mu - \nu)$ at the point R^*. Consequently, the upward-sloping locus of points shown in Figure 1 would correspond to the condition $\mu > \nu$ for this transactions technology.

If monetary authorities do attempt to minimize the welfare costs of inflationary finance for any given revenue requirement, the analysis of this section suggests that there may be an observable systematic correlation between movements in the nominal interest rate and movements in the reserve ratio for countries that rely on the inflation tax. There are some regions of the world where governments do rely on inflation tax revenue and where monetary policy is characterized by an active use of the reserve ratio. For instance, in Latin America virtually all governments modified their central bank charters during and after the Depression to permit flexibility in the use of reserve requirements. By the early 1970s many countries in Latin America had set legal reserve requirements on demand deposits at levels that exceeded 30 percent and frequently modified the reserve requirements in response to fiscal financing requirements.[10]

Colombia, for example, altered the legal reserve ratio on demand deposits seventy-four times between 1952 and 1974, gradually raising the base level of the reserve ratio from 14 percent at the start of the period to 41 percent by mid-1974. The basic legal reserve ratio on demand deposits was frequently supplemented by marginal reserve ratios for new deposits that ranged between 40 and 100 percent. Substantial increases in the reserve ratio were associated with exchange rate crises (from 14 to 23 percent during 1957–58, from 18 to 27 percent during 1962–63, and from 23 to 34 percent during 1967–68) as well as with a period of expansionary government expenditure programs (from 31 to 41 percent during 1973–74).[11]

To determine whether the use of reserve requirements in Latin America and in other parts of the world is systematically related to the inflation rate, Table 1 computes correlations between the average reserve ratio on bank deposits and the inflation rate for forty-one countries during the period 1960–84.[12] The IMF's

[10] Tamagna (1965) contains a detailed description of the legislation in each country that created flexible reserve requirements, as well as a thorough discussion of the uses to which reserve requirements were put.

[11] The information on changes in the legal reserve ratio can be found in the Colombian central bank's *Revista del Banco de la República*, various issues. Díaz Alejandro (1976) is a good source for a description of the exchange rate crises and fiscal policies.

[12] Inflation rates were used because of the absence of information on nominal interest rates. There is a potential pool of 134 countries in the IMF that could have been studied. To reduce the number to a manageable size, countries were chosen that possessed at least twenty years of data and had at least ten million people. The two criteria are arbitrary and would need to be relaxed in a more thorough investigation of the link between fiscal variables and the reserve ratio.

TABLE 1

INTERNATIONAL CORRELATIONS OF THE INFLATION RATE AND RESERVE RATIO

Country	Average Reserve Ratio Mean	(Std. Dev.)	Inflation Mean	(Std. Dev.)	Correlation	(Signif. Prob.)
Latin America						
Argentina (60–84)	.50	(.53)	1.16	(1.52)	.42*	(.0363)
Brazil (60–84)	.35	(.08)	.56	(.44)	−.30	(.1418)
Chile (60–84)	.33	(.16)	.84	(1.32)	.41*	(.0444)
Colombia (60–84)	.29	(.12)	.17	(.09)	.47*	(.0191)
Mexico (60–84)	.35	(.19)	.18	(.24)	.57**	(.0028)
Peru (60–84)	.44	(.15)	.31	(.33)	.53**	(.0067)
Venezuela (60–84)	.23	(.03)	.06	(.06)	−.18	(.3763)
Africa						
Egypt (60–84)	.27	(.07)	.08	(.06)	.04	(.8460)
Ethiopia (61–83)	.23	(.12)	.06	(.07)	.45*	(.0315)
Ghana (60–83)	.32	(.13)	.26	(.30)	.73**	(.0001)
Madagascar (65–84)	.05	(.05)	.10	(.09)	.61**	(.0043)
Morocco (60–84)	.06	(.02)	.06	(.05)	−.46*	(.0218)
Nigeria (60–84)	.17	(.09)	.12	(.11)	.54**	(.0051)
South Africa (60–84)	.07	(.02)	.08	(.05)	−.68**	(.0002)
Sudan (60–83)	.32	(.11)	.12	(.12)	.41*	(.0464)
Zaire (64–84)	.44	(.14)	.38	(.28)	−.05	(.8427)
Asia						
Burma (60–83)	.42	(.40)	.07	(.11)	.66**	(.0005)
India (60–84)	.09	(.02)	.08	(.07)	.16	(.4314)
Indonesia (62–84)	.33	(.17)	1.06	(2.53)	.32	(.1498)
Korea (60–84)	.20	(.08)	.14	(.08)	.02	(.9271)
Malaysia (61–84)	.10	(.02)	.04	(.04)	.10	(.6461)
Nepal (60–83)	.31	(.13)	.07	(.08)	−.21	(.3257)
Pakistan (60–84)	.12	(.02)	.08	(.06)	.21	(.3057)
Philippines (60–84)	.13	(.03)	.11	(.11)	.43*	(.0325)
Sri Lanka (60–84)	.17	(.03)	.07	(.07)	−.20	(.3401)
Thailand (60–83)	.10	(.04)	.06	(.06)	−.40	(.0551)
Middle-Income Europe						
Greece (60–84)	.12	(.04)	.10	(.09)	.02	(.9299)
Portugal (60–84)	.15	(.05)	.13	(.09)	−.50**	(.0107)
Spain (60–84)	.08	(.06)	.11	(.06)	.35	(.0911)
Turkey (60–84)	.29	(.08)	.21	(.25)	.58**	(.0023)
Yugoslavia (60–84)	.29	(.09)	.19	(.13)	.30	(.1436)
Industrialized Countries						
Australia (60–84)	.10	(.03)	.07	(.05)	−.59**	(.0017)
Belgium (60–84)	.02	(.01)	.05	(.03)	−.33	(.1087)
Canada (60–84)	.06	(.02)	.06	(.04)	−.66**	(.0004)
France (60–84)	.04	(.02)	.07	(.04)	.05	(.8004)
Germany (60–84)	.11	(.02)	.04	(.02)	.10	(.6288)
Italy (60–84)	.12	(.03)	.10	(.06)	.78**	(.0001)
Japan (60–84)	.03	(.01)	.07	(.05)	.60**	(.0017)
Netherlands (60–84)	.01	(.01)	.06	(.02)	−.48**	(.0146)
United Kingdom (60–84)	.07	(.03)	.08	(.06)	−.25	(.2279)
United States (60–84)	.08	(.02)	.05	(.04)	−.49**	(.0125)

*Significant at the .05 level.
**Significant at the .05 level.

Source: *International Finance Statistics Yearbook*, 1985, *International Financial Statistics*, April 1986. Inflation rates are given at the beginning of the *IFS* Yearbook and measure the proportional change in the consumer price index from midyear to midyear. Reserve ratios are calculated by the following formula: (14 − 14a)/(34 + 35 − 14a), where the numbers refer to the *IFS* line numbers for the monetary base (14), currency outside of banks (14a), money (34), and quasi money (35). Reserve ratios are end-of-year figures.

International Financial Statistics allows the computation of the average reserve ratio on demand and time deposits but does not contain information on legal reserve ratios. For the computed average reserve ratios, standard money multiplier theory predicts that fixed legal reserve ratios on demand and time deposits will cause the average reserve ratio to decline with an increase in the interest rate. Positive correlations between the average reserve ratio and the inflation rate will therefore reflect monetary policies that actively raise reserve requirements when inflation rates rise.[13]

Table 1 reports information on the inflation rate, average reserve ratio, and the correlation between the two variables over the period from 1960–1984. Table 1 shows that statistically significant positive correlations between the reserve ratio and the inflation rate occur in five of the seven Latin-American countries, in five of the nine African countries, in two of the ten Asian countries (Burma and the Philippines), in one of the five middle-income European countries (Turkey), and in two of the ten industrialized countries (Japan and Italy). Statistically significant negative correlations occurred in none of the Latin-American countries, in two of the African countries (South Africa and Morocco), in none of the Asian countries, in one of the middle-income European countries (Portugal), and in four of the industrialized countries.

One conclusion that emerges from Table 1 is that systematic increases in the reserve ratio accompany increases in the inflation rate in Latin America and Africa. The reserve ratio appears to be less actively used in Asia and in industrialized countries, with the observed negative correlations between the inflation rate and the average reserve ratio partly reflecting changes in the money multiplier. The empirically observed correlations between the reserve ratio and the inflation rate in Table 1 may reflect attempts by monetary authorities to minimize the welfare cost of inflationary finance, given that a certain amount of revenue must be raised by the inflation tax. However, it is worth noting that many developing countries do not possess competitive banking systems of the sort modeled in this paper. To the extent that an oligopolistic banking structure or government monopoly control of the

[13]The average reserve ratio can be expressed as the following, where τ is the legal reserve ratio on demand deposits, θ is the legal reserve ratio on time deposits, t is the ratio of time deposits to demand deposits, and e is the ratio of excess reserves to demand deposits:

$$\tau_{avg} = \frac{\tau + \theta t + e}{1 + t} .$$

Differentiation of the average reserve ratio with respect to the nominal interest rate shows that the correlation between the reserve ratio and the interest rate depends on the three terms shown below:

$$\frac{d\tau_{avg}}{di} = \frac{de/di}{(1 + t)} + \frac{(\theta - \tau - e)\frac{dt}{di}}{(1 + t)^2} + \frac{\frac{d\tau}{di} + t\frac{d\theta}{di}}{(1 + t)} .$$

The first term in the above expression will be negative under the standard assumption that the demand for excess reserves by banks is negatively related to the opportunity cost of holding them. The second term, also, will be negative under the usual assumption that the legal reserve ratio on time deposits is less than the legal reserve ratio on demand deposits and that portfolio switching out of currency and demand deposits into time deposits in response to higher nominal interest rates raises the ratio of time deposits to demand deposits. The general presumption, then, is that optimizing behavior by banks and the public to higher nominal interest rates will tend to lower the average reserve ratio. The third term captures the response of the central bank to increases in the nominal interest rate.

banking system is important in a country, the analytical results of this paper may need modification.[14] Whether the observed correlations between the reserve ratio and the inflation rate in Table 1 are, in fact, the correlations that would correspond to the slopes of the sets of tangency points between iso-revenue and iso-welfare curves must be the subject of future research.

4. CONCLUSION

This paper has developed the analysis of inflationary finance for a production economy in which the government uses the reserve ratio in conjunction with the nominal interest rate to tax currency and demand deposits. Essentially three conclusions emerge from the model. First, the semielasticity of the demand for money with respect to the direct and indirect effects of changes in the nominal interest rate rarely gives an accurate measure of the revenue-maximizing nominal interest rate and may even seriously underestimate the revenue-maximizing nominal interest rate if the government is constrained in its use of the reserve requirement (including the constraint that the reserve ratio cannot exceed 100 percent). In addition, even for an unconstrained government the semielasticity of demand for money will only give an accurate estimate of the revenue-maximizing nominal interest rate if there are no output effects associated with the use of the inflation tax (so that on the margin all time spent on transactions comes out of leisure).

Second, when governments rely on inflationary finance at levels less than the revenue-maximizing amount, the paper shows that the minimization of welfare costs will result in the choice of a combination of a reserve ratio and nominal interest rate that lies on a locus of tangency points formed by the government's iso-revenue curves and the iso-welfare curves that maintain the representative agent at a given level of welfare. Third, the examination of empirical correlations of the reserve ratio and the inflation rate for forty-one countries over the period from 1960 to 1984 shows that a number of governments do actively alter the reserve ratio with a tendency for increases in the reserve ratio to accompany increases in the inflation rate in Latin America and Africa. Although the paper's empirical evidence is not sufficient to determine whether governments do attempt to use reserve requirements to minimize the welfare cost of inflationary finance, the evidence does provide a starting point for a more thorough investigation of the interaction between fiscal deficits, the inflation rate, and the reserve ratio in economies that rely on revenue from the inflation tax.

APPENDIX

The first elasticity expression in (9) is derived by noting that the first-order condition for revenue maximization with respect to the nominal interest rate can be written as

[14]See McKinnon and Mathieson (1981) for a discussion of the use of reserve requirements in developing countries. Fry (1981) examines revenue generation from the inflation tax when the government is a monopoly supplier of currency and deposits.

$$\eta_{mi} \equiv -\frac{i}{m}(m_1 + m_2\tau) = 1 + \frac{\tau d}{m} + \frac{i}{m}\left[m_3\frac{\partial c}{\partial i} + \tau d_1 + \tau^2 d_2 + \tau d_3 \frac{\partial c}{\partial i}\right]. \tag{A1}$$

The derivation is completed by adding and subtracting

$$\frac{h}{m}\eta_{h\tau} \equiv \frac{\tau d}{m} + \frac{1}{m}\left[i\tau m_2 + \tau m_3 \frac{\partial c}{\partial \tau} + i\tau^2 d_2 + \tau^2 d_3 \frac{\partial c}{\partial \tau}\right]$$

from the right-hand side of (A1) noting that $m_2 = d_1$. The other elasticity expressions in (9) and footnote 7 are derived similarly. Equation (12) is derived from the condition $(\partial R/\partial \tau)(\partial U/\partial i) = (\partial R/\partial i)(\partial U/\partial \tau)$ that holds at the points of tangency of the iso-revenue and iso-welfare curves. Since $\partial R/\partial \tau = i(\partial h/\partial \tau)$, multiplying both sides of the tangency condition by τ/hU allows the condition to be rewritten as

$$\frac{h}{m}\eta_{h\tau}\frac{\eta_{Ui}}{\eta_{U\tau}} = \frac{1}{m}\frac{\partial R}{\partial i} = -\eta_{mi} + 1 + \frac{\tau d}{m} + \frac{i}{m}\left[m_3\frac{\partial c}{\partial i} + \tau d_1 + \tau^2 d_2 + \tau d_3 \frac{\partial c}{\partial i}\right]. \tag{A2}$$

Equation (12) results from (A2) by adding and subtracting $\frac{h}{m}\eta_{h\tau}$, as was done in (A1) in deriving (9).

In Figure 1 the $\partial R/\partial \tau = 0$ schedule is defined by the condition that $i = d(\phi_{dd}\phi_{mm} - \phi_{md}^2)c/(\tau\phi_{mm} - \phi_{md})$. For transactions cost functions that are quadratic the $\partial R/\partial \tau = 0$ schedule is bounded on the left by a vertical asymptote at $\tau = \phi_{md}/\phi_{mm}$ and from below by the horizontal axis. The $\partial R/\partial \tau = 0$ schedule is defined by the condition that $i(\partial m/\partial i + \tau \partial d/\partial i) + m + \tau d = 0$, which can be rewritten as

$$i(-\phi_{dd} + 2\tau\phi_{md} - \tau^2\phi_{mm}) + c(\phi_{dd}\phi_{mm} - \phi_{md}^2)(m + \tau d) = 0 \tag{A3}$$

by making use of the derivatives of the asset demand functions. For transactions cost functions that are quadratic, total differentiation of (A3) shows that along the $\partial R/\partial i = 0$ schedule the slope $(di/d\tau)$ is equal to $-[(i)\partial d/\partial i + (1/2)\partial h/\partial \tau]/(\partial h/\partial i)$ so that the schedule reaches its maximum at a reserve ratio that lies between the vertical asymptote of the $\partial R/\partial \tau = 0$ schedule and the revenue-maximizing reserve ratio (where $\partial h/\partial \tau = 0$).

The slope of the iso-welfare curves and the slope of the locus of tangency points depend on the expression $\eta_{Ui}/\eta_{U\tau}$. With a fixed level of consumption (and work effort), leisure is given as follows: $\ell = 1 - \bar{n} - \phi(m, d)\bar{c}$. Therefore,

$$\frac{\partial U}{\partial i} = -U_t c \left[\frac{\phi_m(-\phi_{dd} + \tau\phi_{md}) + \phi_d(\phi_{md} - \tau\phi_{mm})}{\phi_{mm}\phi_{dd} - \phi_{md}^2} \right] \tag{A4}$$

and

$$\frac{\partial U}{\partial \tau} = -U_t c \left[\frac{i\phi_m\phi_{md} - i\phi_d\phi_{mm}}{\phi_{mm}\phi_{dd} - \phi_{md}^2} \right]. \tag{A5}$$

Expressions (A4) and (A5) can be combined to show that

$$\frac{\eta_{Ui}}{\eta_{U\tau}} = 1 + \frac{\phi_m\phi_{dd} - \phi_d\phi_{md}}{\tau(\phi_d\phi_{mm} - \phi_m\phi_{md})} > 1\,. \tag{A6}$$

Equation (A6) can be used to show that the slope of the iso-welfare curves is negative:

$$\frac{di}{d\tau}\Big|_{\bar{U}} = -\frac{i}{\tau}\frac{\eta_{U\tau}}{\eta_{Ui}} = -\frac{i(\phi_d\phi_{mm} - \phi_m\phi_{md})}{\tau(\phi_d\phi_{mm} - \phi_m\phi_{md}) + (\phi_m\phi_{dd} - \phi_d\phi_{md})} < 0\,. \tag{A7}$$

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